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# Acceleration due to gravity variation of the earth with time



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*Content;*

# 1. Abstract;

Acceleration due to gravity is the intrinsic and inherent property of matter, if there is mass then there is gravity. Gravity is nothing but it's just an attraction force acted by systems which are acting continuously on other system which is having unit mass. In fact gravity is described by classical system in which, having a significant mass so that it can influence to other objects too. Acceleration due to gravity of each and every objects depend on the mass and distance of object from the centre of mass of the object which is acting on other body. Same thing is also following by our earth. Our earth is a huge body which is having enormous mass and large radius so that gravity or acceleration due to gravity is held by our earth is in significant amount. Our earth is having significant and affecting acceleration due to gravity and it is around 10 $m/s^2$.

As our earth is formed around 4.56 billion years ago and at that time our earth was highly heated and extreme viscous earth in zig-zag shape. The surface temperature of earth was around $12000^0$C when it was formed from the sun. By the time it became cool down continuous by Stephan's law of radiation consequently upper surface of earth, known is crust was formed in solid state. In the same way the whole volume of the earth became reduce afterwards frequently which volcanic activity played important role for reducing it. Everything is happened by time and in all these phenomena of earth, time played important role. By the time many processes on earth surface are going on in which some physical phenomenon provides significance mass and increasing radius while some physical phenomenon plays role in such a way in which mass of earth convert into energy like decaying of radioactive elements and became escape by time consequently mass of Earth reduces and radius as well.

We are trying to establish some basic idea for indirect effects of time on acceleration due to gravity and evolve some mathematical formula for understanding about the indirect effect of time on acceleration due to gravity of the earth. It may give idea about the increasing and decrasing in accleration due to gravity of the earth surfaces.

# 2. Introduction;

Earth is the third planet of our solar system of the Milky Way galaxy. Earth is densest body among all the planets including sun of this solar system. It is a planet of this solar system where life has completely evolved well in terms of flora and fauna. The earth was formed at around 4.56 billion year before. It was formed by natural explosion of nebula in which all the systems of this solar system were packed. When our earth or other planets was formed they were very hot and in semi fluid state. Probably in the initial stage when it was the part of the sun, they were in the form of gaseous stages and highly messed with the sun due to high acceleration due to gravity of the pre-solar system. According to Nebular Hypothesis the pre-solar system has broken down. By the time these planets were became cold very slowly by Stephan's law of radiation followed by Newton's law of cooling. Same thing was also happened with our earth. Earth was highly dense and huge mass body when it was formed and even today this is densest planet of this solar system. As our earth become denser with time consequently acceleration due to gravity of this earth probably increased. Though the acceleration due to gravity of the earth depends on the mass and radius of Earth. From semisolid to solid stage of the earth, the radius of our earth surely is decreased but we can't make sure about the increased or decreased mass of the earth.



As there is some physical phenomenon which reflects that the little mass of our earth is escape from earth through volcanic activity. In the initial stage of our Earth, volcanic activities was happened frequently. Sometime meteoritic impact was also acted on earth surface which also increase the mass of the earth but insignificance. Still there are many factors which are continuously acting on earth surface and they effect the mass and radius of earth as well.

There are some important factors which are affecting the mass and radius of our earth.

Our earth is near about spherical shape in current era. Sometime it is consider as an ellipsoid more specifically oblate ellipsoid in such a manner of which both horizontal axes is constantly same while third axis is different. When our earth was broken down from solar system, it was highly heated and most expect fully it was in zigzag shape as other systems have broken down. Since all the objects wants to be spherical shape because of tendency for lying in minimum potential. Due to gravitational force of Sun and cooling's effect, our earth has gained spherical shape.

When earth was formed, obviously it was highly viscous and heated, in general it was in fluid state but bound due to internal gravity of earth towards centre. By the time, the earth was cool down by temperature difference with surroundings, consequently the radius of earth became reduce gradually and it was also because of escaping of volatile gases from the earth by volcanic activities. Here It is very clear that the earth is not exactly spherical in this era and of course it happened due to rotation of the earth and its revolution around the sun. One more thing that when our earth was formed it was highly heated around 10000K so that the internal collision of particles was little more than low temperature but not pretty much so that inter collision of particles is no more significant.

As it is completely unknown that what was the actual mass and actual radius of earth when it was formed and when did it become spherical of current era.

## 3. Methodology;

As we know that acceleration due to gravity depends on mass and radius of the spherical body.

Now, in the initial stage of acceleration due to gravity of the earth, considering Earth is as an inertial body. Let the initial mass of the Earth is $M_i$ and initial radius of the Earth was $R_i$.

Acceleration due to gravity of the Earth; It is the amount of force acted by the earth on unit mass of object. According to Newton's law of gravity, each and everybody is attracted to each other towards its centre. If the mass of an object is unit then it's called is acceleration due to gravity. It is called acceleration due to gravity. In case of earth is attracting to other system of unit mass then acceleration due to gravity of Earth.

Here it is consider that our earth is considering as inertial frame of reference and also consider that the shape of our earth in initial stage was spherical.

$$g = \frac{G.M_i}{R_i^2}$$

and the general equation of acceleration due to gravity of the earth is

$$g = \frac{G.M}{R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where, G is the universal gravitational constant.

($G = 6.67\times10^{-11}$ N-m$^2$/kg$^2$).

But in actual sense our earth is non− inertial body and it is moving continuously around the sun and its axis as well. So that the motion of planet is called by motion of revolution and rotation respectively.

Let the rotational angular velocity of the earth is $\omega$ and let the radius of the Earth is R. In this case we are considering earth is a spherical body.

Then the actual acceleration due to gravity in initial state (sometimes is consider as effective acceleration due to gravity) of earth will be as followed by using eqn. (1)

$$g' = g - R_i\omega_i^2 cos^2\,\theta$$

$$g' = \frac{G.M_i}{R_i^2} - R_i\omega_i^2 cos^2\theta \text{ ………………….. (2)}$$

Equation (2), which acceleration due to effective gravity depends on mass of earth, radius of earth, and rotational angular velocity of our earth.

Mass, radius and rotational angular velocity of our earth are variables which also change with time and of course there were change with time from the origin of our earth.

In the equation (2) acceleration due to gravity is the function of mass, radius and angular velocity of the earth as below;

$$g = f(m, R, \omega)$$

$$g = f\{m(t), R(t), \omega(t)\}$$

$$g = \frac{G.M}{R^2} - R\omega^2 cos^2\theta \text{ ……………………………….. (3)}$$

As g is the function of m, R and $\omega$ and they are varying with time t.

Now differentiating eqn. (3) with respect to time t

We get;

$$\frac{d(g)}{dt} = \frac{d(f(m,R,\omega))}{dt}$$

$$\frac{d(g)}{dt} = \frac{d}{dt}\left(\frac{G.M}{R^2} - R\omega^2 cos^2\theta\right)$$

$$\frac{dg}{dt} = \frac{d}{dt}\left(\frac{G.M}{R^2}\right) - \frac{d}{dt}(R\omega^2 cos^2\theta)$$

$$\frac{dg}{dt} = \frac{G}{R^2}.\frac{dM}{dt} - \frac{2G.M}{R^3}.\frac{dR}{dt} - [\omega^2 cos^2\theta.\frac{dR}{dt} + 2.R.\omega.cos^2\theta.\frac{d\omega}{dt}]$$

$$\frac{dg}{dt} = \frac{G}{R^2}.\frac{dM}{dt} - \frac{2G.M}{R^3}.\frac{dR}{dt} - \omega^2 cos^2\theta.\frac{dR}{dt} - 2.R.\omega.cos^2\theta.\frac{d\omega}{dt}]$$

$$\frac{dg}{dt} = \frac{G}{R^2}.\frac{dM}{dt} - [\frac{2G.M}{R^3} + \omega^2 cos^2\theta]\frac{dR}{dt} - 2.R.\omega.cos^2\theta.\frac{d\omega}{dt} \text{ ……… (4)}$$

Equation (4) is the time derivate of acceleration due to gravity of earth in which all the variables change with time, so eqn. (4) is applicable for determining the variation in the value of g by time.

There is three derivatives which is continuously changing with time from the origin of earth to the today in the history of earth and it is expected to change till long.

## 4. Mass of our earth

When our earth was formed it is expectedto relatively less dense from today as it was highly heated and was having highly volatile materials. As the most of the volatile materials escape in to the space consequently it lost its mass. By assuming these things one can say that the amount of mass of the earth was more than today irresepected some other meteoritic phenomenon took place. It is also because of frequently volcanic activities, escaping of light gases like Hydrogen (H), Helium (He) and some other gases time by time by volcanic eruptions. In the initial stage the nuclear fission of radioactive elements was also relatively much than today consequently the mass of the earth reduced in initial stage in very short time comparative to the total age of our earth. As our earth is huge body so that mass loss is not significant visible.

Now a days our earth is lossing it's mass by 50000 tons per year (https://scitechdaily.com/earth-loses-50000-tonnes-of-mass-every-year/).

On the earth there is large number of meteorites fall from the free space towards the earth surface in every second but they are in dust shape and size. The amount of mass of falling meteorites is in around 37000 to 78000 tons per year. (http://curious.astro.cornell.edu/about-us/75-our-solar-system/comets-meteors-and-asteroids/meteorites/313-how-many-meteorites-hit-earth-each-year-intermediate).

Some amount (in weight) of light gases were also escaped from the earth surface for forever due to different causes. Consequently the mass of earth reduced. By cooling the earth, the radius of the earth is also decrease with time.

In both scenario the earth become denser consequently acceleration due to gravity of the earth is increased with time.

Rate of change of mass of the earth is

$$\frac{dM}{dt} = -50000 \times 10^3\ kg/year$$

$$\frac{dM}{dt} = -5 \times 10^7\ kg/year$$

$$\frac{dM}{dt} = -1.585\frac{kg}{sec}.$$

Thus the rate of change of mass of the earth is around 1.6 kg/sec. Since it is negligible but there is some significant value in long duration (in millions of years and play an important role in rate of change of acceleration due to gravity.

## 5. Radius of the earth;

In the initial stage of our earth, it was expected to be relatively big in shape in relative to the size of our earth in zigzag order. It was kind of broken heated stone in no specific shape. By the time it was cool down and now this became in this shape what I actual got. Once it was formed from the sun, it was highly heated as it was external part of the sun and formed in later age of the formation of our solar system so that it was constituted by elements of higher order atomic number relatively than the constituents of the sun as it is now a days.

Due to cooling and shrinking towards the centre of the earth it become in smooth shape by the time in earlier stage of this earth and it was around 4 billion year ago. It was very crucial time for to make stable and at minimum potential of the earth.

In the recent studies, the radius of our earth decreases with time, and it is disclosed by the NASA in 2011.
(https://www.nasa.gov/topics/earth/features/earth20110816.html).

According to the study of NASA, change of radius of the earth is 0.1 mm per year $(10^{-4} m/year)$. Though it is very insignificant value as today but after a long time it gives a significant change in the acceleration due to gravity of the earth. Though there is no more significant reason is there that why radius of the earth varies with time. What types of reason is there for contracting to our earth? Still rate of change of radius of our earth will play important role to change in acceleration due to gravity with time.

Rate of change of radius of our earth is $\frac{dR}{dt} = 10^{-4} m/year$

$$\frac{dR}{dt} = 10^{-4} m/year$$

$$\frac{dR}{dt} = 3.17 \times 10^{-12} m/sec$$

A reduction in the radius and considerable variations in the volume, surface area, mass and average density of the Earth were calculated from amplitudes of the Precambrian and present-day surface relief. Over a period of 4 Billion years, the overall reduction in the Earth’s radius from heat and mass losses was determined to be 52 km or $1.3 \times 10^{-3}$ cm/year. A good correlation between the epochs of global cationization of the earth’s crust and the reduction in the sphere volume was established.

## 6. Angular velocity of our earth;

As our earth is moving about its axis and around the sun continuously as well, these motions are known as rotational angular velocity and revolution angular velocity respectively. The angular velocity of our earth is decreasing from a long time. Our earth is going to slow year by year. Though the rate of change of change of angular velocity is quit less, to significant effect on centrifugal force which in incorporate to reduce effective acceleration due to gravity of the earth.

Our earth complete one rotation about its axis is 23 hours 56 minutes and 4 seconds. The rotation of our earth is slowing with time though the rate is very low still there is some significant values. The day length is going to short day by day.

Angular velocity of the earth is decrease with time and it is very less, the rate of change of daytime is 1.7 millisecond/ century. It can say that the rate of time - period of axis of rotation of the earth increases with 1.7 millisecond per century. As Time period T is the function of time and it varies with it. (Long-Term Variations in the Angular Velocity of Earth's Rotation G. G. Denisov, V. V. Novikov, and A. E. Fedorov)

Variations in day length for all the times of instrumental observation (last 300 years) were analysing the sedata, one can reliably separate a constant decrease the rate for the earth's rotation velocity, which corresponds to an increase in day length by 1.7 $\times 10^{-3}$ sec per century. The rate of decrease in the angular velocity of the Earth's rotation velocity.

Hence,

$$\frac{dT}{dt} = \frac{1.7 \times 10^{-3}}{100} sec/year$$

$$\frac{dT}{dt} = 1.7 \times 10^{-5} sec/year.$$

$$\frac{dT}{dt} = 1.7 \times 10^{-5} sec/year.$$

$$\frac{dT}{dt} = 5.39 \times 10^{-13} sec/sec.$$

Where T is the rotational time period of our earth and it is 23 hours 56 minutes and 4 seconds and it varies with time since long time ago.

We know that

$$\omega = \frac{2\pi}{T}$$

Rate of change of acceleration due to gravity of the earth is

So, $\frac{d\omega}{dt} = \frac{d}{dt}(\frac{2\pi}{T})$

$\frac{d\omega}{dt} = -(\frac{2\pi}{T^2})\frac{dT}{dt}$

Where T is the rotational time -period of the earth.

Hence the rate of change of angular velocity of our earth can be find by below relation.

$$\frac{d\omega}{dt} = -(\frac{2\pi}{T^2})\frac{dT}{dt} \text{ ................................. (5)}$$

Rotational angular velocity of our earth is an important part of effective acceleration due to gravity and it varies from place to place.

Now putting the value of $\frac{d\omega}{dt}$ is equation (4);

We get

$$\frac{dg}{dt} = \frac{G}{R^2}.\frac{dM}{dt} - [\frac{2G.M}{R^3} + \omega^2 cos^2\theta]\frac{dR}{dt} - 2.R.\omega.cos^2\theta.(-(\frac{2\pi}{T^2})\frac{dT}{dt})$$

$$\frac{dg}{dt} = \frac{G}{R^2}.\frac{dM}{dt} - [\frac{2G.M}{R^3} + \omega^2 cos^2\theta]\frac{dR}{dt} + \frac{4\pi R.\omega cos^2\theta}{T^2}\frac{dT}{dt} \text{ ......... (6)}$$

In the above relation, there is three things which varies with time and it is $\frac{dM}{dt}$, $\frac{dR}{dt}$ & $\frac{dT}{dt}$ are varies with time.



Equation (6) is valid for right scenario but probably this relation will be valid for long past. It is also because there were many other physical phenomena were happening which are not now a days in spherical earth.

For solving this problem, we may use equation (6) as given below;

$$\frac{dg}{dt} = \frac{G}{R^2}.\frac{dM}{dt} - [\frac{2G.M}{R^3} + \omega^2 cos^2\theta]\frac{dR}{dt} + \frac{4\pi R.\omega cos^2\theta}{T^2}\frac{dT}{dt}$$

By putting values of several variables in above equation we can find rate of change of acceleration due to gravity.

$\frac{dM}{dt} = -5 \times 10^7 \frac{kg}{ye}$

$\frac{dM}{dt} = -1.585 \frac{kg}{sec}$.

Rate of change of radius

$$\frac{dR}{dt} = 10^{-4} m/year$$

$$\frac{dR}{dt} = 3.17 \times 10^{-12} m/sec$$

$$\frac{dT}{dt} = 5.39 \times 10^{-13} sec/sec.$$

Now, putting these values in above equation

$\frac{dg}{dt} = \frac{6.67\times10^{-11}}{\{(6.678)^{6}\}^{2}}.(-1.587) - [\frac{2.(6.67\times10^{-11})\times5.98\times10^{24}}{(6.67\times10^{6})^{3}} + (2.23 \times 10^{3})^{2}cos^{2}\theta](3.17\times10^{-12}) + \frac{4\pi(6.378\times10^{6})2.23\times10^{3}.cos^{2}\theta}{T^{2}}(5.39\times 10^{-13})$

$\frac{dg}{dt} = -\frac{10.585\times10^{-11}}{\{(44.6\times10^{12}\}} - [\frac{7.97\times10^{14}}{2.97\times10^{20}} + 4.97\times10^{2}cos^{2}\theta](3.17\times10^{-12}) + \frac{9.62\times10^{-2}.cos^{2}\theta}{7.46\times10^{9}}$

$\frac{dg}{dt} = -2.37\times10^{-24} - [2.68\times10^{-6} + 4.97\times10^{2}cos^{2}\theta](3.17\times10^{-12}) + 1.29\times10^{-11}cos^{2}\theta$ …………………………………………….. (14)

Equation (14) depends on latitude too. Let us consider for 0 degree latitude.

Cos(0) = 1

Then from equation (14),

$\frac{dg}{dt} = -2.39\times10^{-24} - [8.49\times10^{-18} + 15.75\times10^{-10}] + 1.29\times10^{-11}$

$\frac{dg}{dt} = -2.39\times10^{-24} - [15.75\times10^{-10}] + 1.29\times10^{-11}$ ……………... (15)

Since sum of one big number with relatively very small no. will be equal to big number so that

$8.49\times10^{-18} + 15.75\times10^{-10} = 15.75\times10^{-10}$

Now from equation (15),

$\frac{dg}{dt} = -2.39\times10^{-24} - [1.575\times10^{-9}] + 1.29\times10^{-11}$

$\frac{dg}{dt} = -1.575\times10^{-9}$

Since net value of $-2.39\times10^{-24} - [1.575\times10^{-9}] + 1.29\times10^{-11}$ will be nearly equal to $-1.575\times10^{-9}$

$\frac{dg}{dt} = -1.575\times10^{-9}$

Thus the rate of change if acceleration due to gravity of the earth is negative, hence it is clear that the acceleration due to gravity of the earth is losing, consequently strength to attract some something is losing. As the acceleration due to gravity depends on three parameters in with rate of change of mass and rate of change of radius of earth is insignificant. While rate of change in angular velocity of earth is relatively more significant. Thus, the value become relatively stable.

## 7. Conclusions;

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Since our earth is around 4.6 billion years old and the situation on those days, is no more predictable in terms of strength of earth to attract of something. But by the passes of time it became cool down and probably it was shrink. The situation in now a days if we go through the data what it is based its changes.

Acceleration due to gravity of the earth decreases with the rate of $\mathbf{1.575 \times 10^{-9}} \frac{\boldsymbol{N}}{\boldsymbol{kg}} - \boldsymbol{sec}. or\ \mathbf{1.575 \times 10^{-9}}$ m/sec$^{-3}$. Since no one can say that similar pattern follows all the time because there is not a uniform situation all the time to either reduce or increase it. Since the rate of change of acceleration due to gravity is that much what can't feel in short duration but it might take fine significant values after thousands of years.

The rate of change of acceleration due to gravity of reflects that it decreases in current era due to several time depending factors. The rate of change of accelerating due to gravity reflects that it is minutely changes with time so its significant effects on other related phenomenon can observe in long period of time. There is a many physical phenomenon that are affecting by acceleration due to gravity of the Earth, so its effect can be observe on that after a thousands of year.